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ABSTRACT

This paper reports from a research project at Oxford in the UK that focused (a) on university mathematics teachers' conceptualizations of first-year undergraduate teaching related to observation of their teaching; and (b) on issues relating the conceptualizations to mathematics as a discipline. This research builds on a qualitative study of learning difficulties of first year undergraduates in their encounter with the abstractions of advanced mathematics within a tutorial-based pedagogy. Six tutors' responses to and interpretations of such difficulties were studied in semi-structured interviews conducted during an 8-week university term and following minimally-participant observation of their tutorials. This paper describes a 4-stage spectrum of pedagogical development (SPD) that emerged from the analysis of the tutors' conceptualizations of the students' difficulties, descriptive accounts of the strategies they employ in order to facilitate their students' overcoming of these difficulties, and self-evaluative reflective accounts regarding their teaching practices. This study then exemplify the third and fourth stages of SPD thought a discussion of characteristic examples from the interview data. In these stages the tutors' strategies begin to resemble less a traditional induction process and more a process of facilitating the students' construction of mathematical meaning. This discussion employs tools from sociocultural, enactivist and constructivist theories on the teaching and learning of mathematics. In particular, the data used here exemplify certain tutor strategies to achieve consideration of students' needs. (Author)

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DEVELOPING A PEDAGOGIC DISCOURSE IN THE TEACHING OF UNDERGRADUATE MATHEMATICS: On Tutors' Uses of Generic Examples and Other Techniques

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ABSTRACT

This paper reports from a research project at Oxford in the UK that focused (a) on university mathematics teachers' conceptualisations of first-year undergraduate teaching related to observation of their teaching; and (b) on issues relating the conceptualisations to mathematics as a discipline. This research builds on a qualitative study of learning difficulties of first year undergraduates in their encounter with the abstractions of advanced mathematics within a tutorial-based pedagogy. Six tutors' responses to and interpretations of such difficulties were studied in semi-structured interviews conducted during an 8-week university term and following minimally-participant observation of their tutorials.

We describe a 4-stage spectrum of pedagogical development (SPD) that emerged from the analysis of the tutors'

1. conceptualisations of the students' difficulties;
2. descriptive accounts of the strategies they employ in order to facilitate their students' overcoming of these difficulties; and,
3. self-evaluative reflective accounts regarding their teaching practices.

We then exemplify the third and fourth stages of SPD with regard to (2) through a discussion of characteristic examples from the interview data. In these stages the tutors' strategies begin to resemble less a traditional induction process and more a process of facilitating the students' construction of mathematical meaning. In our discussion we employ tools from sociocultural, enactivist and constructivist theories on the teaching and learning of mathematics. In particular, the data used here exemplify certain tutor strategies such as: *encouraging the students' use of **rich and evocative verbal descriptions** of mathematical concepts, properties and relationships; using **generic examples** and offering **genetic decompositions** to create and reinforce concept images of newly introduced concepts; highlighting the **transferability of a technique** rather than dwelling on mastering its execution; employing **empathetic methods** (pretend ignorance of sophisticated methods) to achieve consideration of students' needs.*

Overall we propose SPD as a useful pedagogic descriptor of undergraduate mathematics teaching.

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1. Rationale and Theoretical Perspectives

In the UK and other countries, in recent years there has been a number of changes that have affected the teaching of mathematics at university level: the number of students attending university has increased while the number of students opting for mathematically-oriented studies is decreasing (Holton et al 2001); recruitment of good mathematics graduates to mathematics teaching is at an all-time low; profound changes have taken place in secondary education pedagogy and curriculum; the gap between secondary and tertiary mathematics education regarding teaching approaches has increased substantially (LMS, 1995); the rapid development of information technology has affected educational practice in the use of computers and calculators in mathematics instruction; finally there has been an increasing demand from universities to be accountable to society regarding, in particular, the quality of their teaching. Moreover, despite the response so far to these changes being mostly towards modifications of the university mathematics curriculum to adjust to the skills of the new intake of students (Kahn & Hoyles, 1997), there is an emerging realisation that reform should be focusing on teaching (Jalling & Carlsson, 1995). The above imply that there is a need for a revision of the underlying principles as well as the practices with regard to the teaching of mathematics at university level (HEFCE, 1996) and that this revision may need to go beyond the extensive, curriculum-based literature in this area, mainly in North America, focusing on central topics such as Calculus (e.g. Ganter, 2000) and Linear Algebra (e.g. Leron & Dubinsky, 1995). Further, and given the often strained relationships between mathematicians in mathematics departments and their colleagues in mathematics education, research that builds the foundations of collaboration between university mathematics teachers and mathematics educators is crucial and, given the pressure currently exercised on universities regarding the need for a scrutiny of their teaching practices, timely. The research project we draw on in this paper aimed at contributing in this area.

Given this state of affairs pedagogical research involving the undergraduate mathematics teacher is limited (e.g. see (Burton and Morgan, 2000). Indeed the research on teacher thinking processes that has informed our study is largely located in the secondary sector (e.g. Brown and McIntyre 1993; Jaworski 1994). In the words of Brown and McIntyre this influence can be described as 'making sense of teaching from the perspective of teachers themselves'; 'how they construe and evaluate their own teaching, how they make judgements, and why in their own understanding, they choose to act in particular ways in specific circumstances to achieve their successes' (p1). This theoretical perspective is relatively new (until the 50s the focus was mostly on the *didactics* of particular *topics* and in the 50s and 60s teachers' classroom *actions* also attracted research focus; it was in the 70s that researchers realised the necessity also for a systematic study of teachers' *thinking*). Since then several models that attempt to describe teachers' thinking processes have emerged (see for example Brown and McIntyre (1993) and Morine-Dershimer (1990) for relevant reviews).

Often however these attempts suggest 'deficit' models of teaching: in interviews, for example, expert teachers tend to focus on atypical situations of their teaching perhaps because they perceive most of their classroom actions as so ordinary and so obvious as not to merit any comment. As a result, the researchers' attention too tends to be directed mostly towards the *problems* (rather than *achievements* of their craftsmanship) which teachers experience and which they often choose to discuss. This 'deficit' model of teaching is unsatisfactory: innovation needs to take account of what is already being done in classrooms. Moreover no evaluation of teaching can be valid in the absence of extensive and systematic observation of actual teaching and of knowledge on how

teachers conceptualise their own teaching. The study we report here seeks to explore the professional *craft* knowledge of undergraduate mathematics teachers - allowing, hopefully, some space for the 'bonus' of what Brown and McIntyre (1993) call 'teachers' flashes of artistic genius'.

Fundamentally, this research has tried to gain insights into the undergraduate mathematics teacher thinking processes through the complementary lenses of the following three theoretical perspectives (for more detail see (Nardi, Jaworski and Hegedus, submitted):

- Sociocultural theory, particular its enculturative dimension in which participants in a social community are seen to be drawn into the language and practices of the community and to develop knowledge through communication and practice (e.g., Vygotsky, 1962; Lerman, 1996; Wenger, 1998);
- Constructivist theory, particularly its account of individual sense-making of experience, and related cognitive models and structures that describe and explain the construction of knowledge (e.g., Cobb, 1996; Confrey, 2000)
- Enactivist theory, particularly its aspect of codetermination, in which living beings and their environment are seen to stand in relation to each other through mutual specification or codetermination (e.g., Dawson, 1999; Varela et al, 1991; Kieren, 1995)

We now briefly introduce the methodology of the *Undergraduate Mathematics Teaching Project* (UMTP) – for more detail see (Jaworski, Nardi and Hegedus, submitted).

2. UMTP and the Spectrum of Pedagogical Development

The Undergraduate Mathematics Teaching Project is a one-year qualitative study funded by the Economic and Social Research Council in the UK and was motivated by an earlier study of undergraduate tutorials (Nardi, 1996) which indicated the richness of the tutorial context in learning *and* teaching incidents. Participants were six experienced mathematicians who acted as tutors to first year undergraduates. Data collection took place over one university term (8 weeks - a third of the academic year) with one member of the research team observing one or two tutorials (each of one hour) for each tutor per week, and conducting one half-hour interview per week related to the tutorial(s). Thus, data consisted of about 75 hours of audio-recordings from tutorials, plus associated field notes, plus 45 audio-recorded interviews each of 30-45 minutes, transcribed fully.

The questions for the semi-structured interviews were directly related to instances from the observed tutorials and to the theoretical perspectives of the researchers. The analysis of the interview data, drawing from data-grounded theory techniques (Glaser & Strauss, 1967), was initiated by the construction of interview protocols: factual summaries of the interview contents. Two levels of coding were undertaken, one mathematical focused and one pedagogically focused. The most commonly occurring pedagogical codes were found to be:

REC STU PRO	Recognition of and reaction to students' problems, needs and abilities.
TUT OBJ STU LEA	Tutor's objectives for students' learning.
TUT MATH STR STU	Identifying mathematical strategies for students.
DIFF TUT HELP STU	Tutor's difficulties in deciding on an approach to help students overcome difficulty.

Alongside the coding process, 82 significant episodes were extracted from the data, approximately two from each week for each tutor. These episodes were set against the most

commonly occurring codes and a subset of 32 episodes was found in which these codes were most evident. Further analysis of the 32 episodes was undertaken and presented in tabulated format that included: details on the episode such as its duration and position on the recording, name of tutor, mathematical content and associated codes; a brief description of the episode content; the fully-transcribed part of the interview that constituted the episode; and an analytical account. Scrutiny of the analytical accounts across the 32 episodes led to identification of themes which in turn led to what we call a 'spectrum of pedagogic awareness or development' which is the focus of this paper.

The 4-stage *Spectrum of Pedagogical Development* (SPD). The *spectrum of pedagogic awareness*, or *development* emerging from this research sought to capture aspects of tutors' pedagogical thinking as expressed through their articulation of teaching issues in the interviews. There seemed to be a number of levels of awareness which were captured under four headings, forming a progression or spectrum as follows:

I. Naive and Dismissive: acknowledging ignorance of pedagogy; recognition of student difficulties with little reasoned attention to their origin or to teaching approaches that might enable students to overcome difficulty.

II. Intuitive and Questioning: involving implicit and hard to articulate but identifiable pedagogic thinking; recognition of student's difficulties with intuition into their resolution, and questioning of what approaches might help students.

III. Reflective and Analytic: including evidence of awareness in starting to articulate pedagogic approaches and of reflection enabling making strategies explicit; clearer recognition of teaching issues related to students' difficulties and analysis of possibilities in addressing them.

IV. Confident and Articulate: involving considered and developed pedagogic approaches designed to address recognised issues; recognition and articulation of students' difficulties with certain well-worked-out teaching strategies for addressing them; recognition of issues and critiquing of practice.

We use the term 'spectrum' to indicate a sense of continuum, with sharp points. Episodes might fit neatly into a category but, more typically, characteristics would shade between categories. We also need to emphasise that these are not categories of teacher or tutor. They reflect particular teaching events or approaches: different tutors exhibited different characteristics at different times. The nature of the research, in asking tutors about their teaching, encouraged (or maybe even required) tutors to reflect on their teaching. Research has shown that such encouragement leads to teachers taking a more questioning, enquiring and articulate attitude to their teaching (Jaworski, 1994). We recognise, therefore, that the pedagogic articulation and development we report are to some extent outcomes of the research itself.

3. Exemplification and discussion of SPD Stages III and IV

Analysis, discussion and exemplification of the data was arranged along three strands that have emerged from the typically recurring codes in the 32 episodes as follows: REC STU PRO underpins Strand 1 (the tutors' conceptualisations of the students' difficulties); TUT OBJ STU LEA and TUT MATH STR STU underpin Strand 2 (the tutors' descriptive accounts of their practices with regard to these difficulties) and DIFF TUT HELP STU underpins Strand 3 (the tutors' self-reflective accounts regarding these practices). Here, through two characteristic

examples, we exemplify the third and fourth stages of SPD along Strand 2 (for a more panoramic presentation see Nardi, Jaworksi and Hegedus (submitted) where we present 12 characteristic examples, four within each strand and where also each example is preceded by a brief description of the larger pool of examples from which it has been drawn). Note: we indicate where a part of a sentence in the transcript has been omitted with […].

Strand 2 The tutors' descriptive accounts of their practices

If formalisation and abstraction are respectively the driving force and the aim of official mathematical communication, materialised on the basis of a number of conventions that are characteristic of the formal mathematical culture, then their adoption is synonymous with a learner's advanced mathematical enculturation (Sierpinska 1994). This process of enculturation may take place with varying degrees of responsibility and ownership between the tutor and the learner. Stages I-IV are described here in terms of these degrees and exemplify tutor practices within Stages III and IV.

At Stage I the tutors perceive their role as being in charge of enculturation. In Hall's terms (Sierpinska 1994) the learner's mathematical enculturation is seen as taking place at the 'informal level': through the accumulation of mathematical experience shared with the expert, the tutor, and through appropriation of the expert's cultural practices. These cultural practices constitute the new-to-the-students habitat of mathematics. The incidents here suggest that the tutors, while recognising the students' difficulty with adopting these practices, appear apprehensive or unaware about the role of teaching in overcoming this difficulty.

At Stage II, the attempts at the enculturation exemplified in the incidents at Stage I, are more focused and more organically informed by the students' needs. In most of the incidents here, the tutors elaborate students' difficulties and employ this elaboration to justify their pedagogical strategies. These strategies include: facilitating the students' resorting to the familiar, previously established knowledge; disentangling students' misconceptions through exposition of correct definitions; enculturating students into the importance and uses of formal mathematical notation and language; enculturating students into the importance and necessity of formal mathematical proof; demonstrating and developing an arsenal of techniques to be used in establishing formal mathematical arguments, e.g. in the context of convergence of sequences and series; suggesting mathematical arguments which optimise the ones suggested by the students; highlighting the epistemological significance of newly introduced concepts, e.g. the concept of coset. Engaging the students in this enculturation process is implied in the tutors' intentions but enacted only to a limited extent.

At Stage III the attempts at the enculturation exemplified in the incidents at Stages I and II, begin to resemble more a process of facilitating the students' construction of mathematical meaning than an induction process. The tutors here openly consider the students' learning and this consideration informs directly their pedagogical practice. The strategies suggested by the tutors here include: disentangling misconceptions through thorough scrutiny of the students' written responses; supporting the construction of mathematical meaning via highlighting the usefulness of verbally describing concepts, properties, relationships etc while remaining alert to what does not carry across from language to mathematics; establishing the importance (necessity and relevance) of formal mathematical reasoning (various ad hoc practices are suggested); coping with the students' reluctance to apply formal definitions (various ad hoc practices are suggested); encouraging the identification of patterns; strengthening students' perseverance on solving a problem by contrasting (under)evaluations of their own work and their actual progress on the problem as well as by providing problem-solving 'tips' (various tips are suggested); determining content of the tutorial on

a carefully balanced combination of pragmatic, pedagogical, epistemological and cognitive grounds; using generic examples to create and enrich concept images of newly introduced concepts.

Example 2.III (Strand 2, Stage III): Using generic examples to create and enrich concept images of newly introduced concepts. Amongst the most discussed strategies that the tutors use in order to assist their students' concept image construction (Vinner & Tall 1981) is the use of examples that embody the essential features of the newly introduced concepts. This has been observed to be a central function of the majority of tutorials as opposed to the more definition oriented, condensed character of the lectures. For example: in the following extract the tutor discusses the role of generic examples in the context of newly introduced topological concepts such as open and closed set of a metric space:

> Tutor: ...as a tutor you're in a position where *[pause]* you know what the relevant examples are which spell out every pitfall and [...] you want to present them with an example which contains all the *[pause]* relevant, um, features and, and phenomena. So you don't want to give an example and say this is your typical open set or something, 'cause it might give them loads of prep- misconceptions about things and so, but *[in this case]* it was a good opportunity to do that. The fact they asked me about metric spaces gave, gave me a chance to explain, you know, the difference between an open and not, um, sorry, not-open and closed and, and er *[pause]* to see why it's not a crazy thing to think of, you know, the closed interval zero one as being open in itself [...] And, but it's actually very important [...] to show that the zero, one closed is open *[pause]* Doesn't look very open if you sit in R^2. [...] it's just, it's just a feature of the space you're working in. I think that's, that's the only problem they'll have in metric spaces. I think that's the standard problem that all undergraduates have is, they always, they always have, they carry this baggage with them like in every other subject, you're trying to remove the baggage and make them *[pause]* think in the way you want them to think. And the baggage they carry into metric spaces, the intuition, the trick's there, it's the ambient space, they all work in the bloody ambient space!

In the above passage on learning-as-construction, the tutor explores the incessant state of conflict and accommodation his students' concept images appear to be in. In particular the chosen example from Topology incorporates significant linguistic and geometric elements that are known to exert strong influence on students' understanding of new topological concepts (Dubinsky & Lewin 1986). The account is significantly strengthened by certain vivid metaphorical associations – such as 'doesn't look very open if you sit in R^2' which seems to allude to a physical embodiment of mathematical ideas (an area of investigation which is currently under vigorous development in works such as Lakoff & Nunez 2000).

At Stage IV the tutors' pedagogical strategies are strongly determined by their intention to engage the students with their own learning and make them active participants in the construction of new mathematical meaning. These strategies include: facilitating the students' construction of new concepts; facilitating the students' acceptance and enactment of formal mathematical proof; enabling students to disentangle misconceptions; suggesting mathematical arguments which optimise the ones suggested by the students; highlighting the transferability of a technique rather than dwelling on mastering its execution; enabling students to overcome the inefficiency of a

compartmentalised view of mathematics; devolving responsibility for learning; employing empathetic methods (pretend ignorance of sophisticated methods) to achieve consideration of students' needs (see Jaworski, Nardi & Hegedus 1999 for further elaboration); offering genetic decompositions (Dubinsky & Lewin 1986) of new mathematical concepts.

Example 2.IV (Strand 2, Stage IV): Overcoming the inefficiency of a compartmentalised view of mathematics. Having observed their students' attempts at problem solving often being severely curtailed by the compartmentalisation of the university mathematics course in deceptively distinct topics (in our data the tutor quoted in this Example elaborated on the potentially damaging effect of this compartmentalisation) the tutors perceive the overcoming of this inefficiency as a major part of their role. In another Example (under Strand 1, Stage III), the tutor, engrossed by her students' convoluted attempts at a question, where a substitution from Analysis would have provided a one-line answer, she referred to the possibility of seeing parts of Probability Theory in conjunction with parts of Analysis, under the wider umbrella of Measure Theory. In the following extract she expands on her role to alter this compartmentalising attitude:

> Tutor: ... the analogy of integration is interesting because I always try to convince them that summation and integration are really the same thing. Because they are, it's just Measure Theory. Um, but it makes life much easier if they can think of sums as integrals and so I do tend to try to do the two together. *[explains the details of doing so in the particular Probability question]* And we'll come back to it when they have to do it again. And they will see this again. This is something that comes up all the time but they've now got the idea and they can worry about it a bit. [...] I mean constantly trying to do these links.

This statement epitomises one of the main characteristics of this tutor's teaching practice, the necessity to make links between mathematics in other courses and links within a single course itself. Later in the interview, in a part omitted here, she exemplifies the methods she employs to pursue this objective: she uses what she calls 'leading diagrams'. The tutor's main aim in making links is to develop a mathematical awareness that enables the student to fit all the pieces of the jigsaw (Analysis, Probability etc.) together.

4. SPD as a useful pedagogic descriptor of undergraduate mathematics teaching

The evidence from UMTP supports *the value of reflection within practice* (e.g. Schon 1987; Brown and McIntyre 1993). In general the tutors' response regarding the significance of this observation-interviewing process as a means of triggering immediate and long term valuable reflection was overwhelmingly positive. Evidence of this was extracted from the parts of the interviews coded as SIGN Q (Tutor highlights a significant event from this week's tutorials) and UMTP METH (The tutor makes an evaluative comment regarding the observation-interviewing procedure of UMTP) – see Nardi, Jaworski and Hegedus (submitted) for a detailed exemplification). This evidence suggests that the explicit intentions of UMTP to engage the tutors in a non-deficit discourse on their pedagogical practice were being conveyed, at least as the data collection period was evolving. The majority of the comments were reflective / pro-active. Can we see then in this self-reflective, pro-active process - implemented in the context of UMTP as a part

of the research process - the seeds of a pedagogy for undergraduate mathematics teaching? Can we see, in other words, an undergraduate mathematics teacher's development as *the route from Stage I to IV of the Spectrum of Pedagogical Development?*

As UMTP explored the pedagogical practices of the tutors from an explicit non-deficit point of view, the fruits of this exploration were remarkable. The tutors' perceptions of student learning and practices could often be embedded in the findings of current cognitive and educational research (see examples in (Nardi, Jaworski and Hegedus, submitted)). This embedding could also be made with regard to the pedagogical strategies employed or suggested by the tutors (such as the use of generic examples in Example 2.III). We wish to propose that the relationship between reflection-in-practice (as provided here by the tutors) and the findings of educational theory (the concept-based research works in the area of advanced mathematical thinking and the sociocultural, enactivist and constructivist theories that were the lenses through which learning and teaching were explored in UMTP) can be a strong one.

What UMTP provided was a context in which educational theory could emerge from a close observation of practice but also a context in which tutors' practice could be informed by an intensive exercise of self-reflection. The claim here is not that in the tutors' struggle to express their perceptions of pedagogic issues (with all the ums, ers, and repetitions) the articulated insights and issues have not been thought about by educators or researchers, but that these are genuine insights for tutors who have given little thought to pedagogy previously. This was an opportunity for the inception and growth of pedagogic ideas – not as a revelation to the mathematics education community – but in demonstration of an evolving growth of awareness of mathematicians and tutors about pedagogy. We might thus suggest that what we report here are insights into how tutors just begin to be reflective on their practice, and how discussions with educators can facilitate this process. So we have here not only findings from a research project, but indications of a way ahead in encouraging pedagogic growth in teacher/tutor development in university mathematics teaching.

Having focused intensively on the elements of effective practice in the tutors' teaching, our findings directly point at the potential of the above outlined dialectic relationship. More action-oriented research in this area is needed in order to substantiate this potential.

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